the Relation Between Technique of Conformal Flat and Damour-Ruffini-Zhao's Method

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Abstract

The relation between the technique of conformal flat and Damour-Ruffini-Zhao's method is investigated in this paper. It is pointed out that the two methods give the same results when the metric has the form $g_{\alpha\beta=0}$, with $\alpha = 0, 1$ and $\beta = 2, 3$. It is indicated that the two methods are not equivalent for general case.

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1 Introduction

There are some schemes to research Hawking radiation for black holes[1][2][3][4][5] [6]. Zhao et al. developed an technique to rapidly determine the location and temperature of event horizon called conformal flat[7][8] . Later they applied the technique to non-stationary black hole[9][10][11]. The technique require that in tortoise coordinate the metric of two dimensional subspace x^0, x^1 is conformal to two dimensional Minkowski space. This requirement does simply and rapidly determine the location and temperature of the horizon simultaneously. This technique is closely related to Zhao's another method[12][13][14][15][16][17], which requires the Klein-Gordon equation has the standard form of wave equation. Therefore there naturally arise a problem: whether or not the two schemes give the same results. Our answer is affirmative for some special case. But for a general case our answer is negative. In the next section we will

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investigate this issue in detail and prove the equivalence between the two schemes when $g_{\alpha\beta=0}$, with $\alpha = 0, 1$ and $\beta = 2, 3$. In section three we illustrate that the two methods is not equivalent for general black hole.

2 the Special Case the Two Scheme Is Equivalent

Consider the case that the metric is

$$g_{\mu\nu} = \begin{bmatrix} g_{00} & g_{01} & 0 & 0 \\ g_{10} & g_{11} & 0 & 0 \\ 0 & 0 & g_{22} & g_{23} \\ 0 & 0 & g_{32} & g_{33} \end{bmatrix} \tag{1}$$

ie

$$g_{\alpha\beta} = 0, \quad with \ \ \alpha = 0, 1 \ \ and \ \ \beta = 2, 3. \tag{2}$$

For the horizon $\xi = \xi(x^0)$, the tortoise transformation is

$$x_*^1 = x^1 + \frac{n}{2\kappa} ln(x^1 - \xi) \tag{3}$$

with other components of coordinates invariant. Therefore in the new coordinates (x^0, x_*^1, x^2, x^3),

$$\tilde{g_{\alpha\beta}} = \frac{\partial x^\lambda}{\partial \tilde{\alpha}} \frac{\partial x^\rho}{\partial \tilde{\beta}} g_{\lambda\rho} = g_{\alpha\beta} = 0, \tag{4}$$

where $\alpha = 0, 1$ and $\beta = 2, 3$ and (2)(3) has been used to obtain the Eq. (4). Therefore only $0, 1$ components of metric change.

$$\tilde{g_{\mu\nu}} = \begin{bmatrix} \tilde{g_{00}} & \tilde{g_{01}} & 0 & 0 \\ \tilde{g_{10}} & \tilde{g_{11}} & 0 & 0 \\ 0 & 0 & \tilde{g_{22}} & \tilde{g_{23}} \\ 0 & 0 & \tilde{g_{32}} & \tilde{g_{33}} \end{bmatrix}. \tag{5}$$

In the two-dimensional subspace (x^0, x_*^1),

$$ds^2 = \tilde{g_{00}} dx^0 dx^0 + \tilde{g_{11}} dx_*^1 dx_*^1 + other \ \ terms = \tilde{g_{00}} [dx^0 dx^0 + \frac{\tilde{g_{11}}}{\tilde{g_{00}}} dx_*^1 dx_*^1] + other \ \ terms. \tag{6}$$

The technique of conformal flat require

$$\frac{\tilde{g_{11}}}{\tilde{g_{00}}} = -1, \tag{7}$$

While he Damour-Ruffini method gives the condition as

$$\frac{g^{\widetilde{11}}}{-g^{\widetilde{00}}} = 1. \tag{8}$$

From(5), one can easily obtain

$$g^{\widetilde{00}} = \frac{g_{\widetilde{11}}}{\tilde{g}_s} \tag{9}$$

$$g^{\widetilde{11}} = \frac{g_{\widetilde{00}}}{\tilde{g}_s}, \tag{10}$$

where $\tilde{g}_s$ is the determinant of submatrix of (5)

$$\tilde{g}_s = \begin{vmatrix} g_{\widetilde{00}} & g_{\widetilde{01}} \\ g_{\widetilde{10}} & g_{\widetilde{11}} \end{vmatrix}. \tag{11}$$

At once Eq.(5) ensure the equivalence between Eq.(7) and Eq.(8).

Next we investigate the equivalence in Eddington coordinates

$$ds^2 = g_{00}dx^0dx^0 + 2g_{01}dx^0dx^1 + g_{11}dx^1dx^1 + (other \quad terms). \tag{12}$$

After take tortoise coordinate transformation, ones obtain in the new coordinate ds^2

$$ds^2 = g_{\widetilde{01}}\{\frac{g_{\widetilde{00}}}{g_{\widetilde{01}}}dx^0dx^0 + 2dx^0dx^1_*\} + other \quad terms. \tag{13}$$

The generalized conformally flatization requires

$$\frac{g_{\widetilde{00}}}{g_{\widetilde{01}}} = -1. \tag{14}$$

In the other hand,

$$g^{\widetilde{00}} = \frac{g_{\widetilde{11}}}{\tilde{g}_s}, \quad g^{\widetilde{01}} = -\frac{g_{\widetilde{01}}}{\tilde{g}_s}. \tag{15}$$

The Klein-Gordon equation in x^1_*, x^0

$$\frac{\tilde{g}^{11}}{\tilde{g}^{01}}\frac{\partial^2}{(\partial x^1_*)^2}\Phi + 2\frac{\partial^2}{\partial x^0 x^1_*}\Phi + (others) = 0. \tag{16}$$

Damour-Ruffini-Zhao's method require the coefficient of $\frac{\partial^2}{(\partial x^1_*)^2}$ being 1

$$\frac{g^{\widetilde{11}}}{g^{\widetilde{01}}} = 1. \tag{17}$$

From Eq.(15) and Eq.(17), Eq.(14) is obtained at once. Therefore in Eddington coordinates, the technique of conformal flat is equivalent to Damour-Ruffini-Zhao's method.

3 Technique of Conformal Flat in General Metric

We will prove that there does not exist the equivalence for general case.

In the previous section, we focus on the special case that the event horizon $\xi = \xi(x^0)$. In this subsection we analyze the general case that $\xi = \xi(x^0, x^2, x^3)$ in Eddington coordinates.

Suppose $x^1 = \xi(x^0, x^2, x^3)$,. Tortoise coordinates transformation,

$$x_*^1 = x^1 + \frac{1}{2\kappa} ln(x^1 - \xi) \tag{18}$$

with other components invariant.

$$dx_*^1 = (1 + \frac{1}{\epsilon})dx^1 - \frac{\xi'_\nu}{\epsilon} dx^\nu, \tag{19}$$

in which $\epsilon = 2\kappa(x^1 - \xi)$ and $\xi'_\nu = \frac{\partial \xi}{\partial x^\nu}$.

The metric is then obtained in terms of tortoise coordinates

$$ds^2 = (\frac{\epsilon g_{11}\xi'_0}{(1+\epsilon)^2} + \frac{\epsilon g_{10}}{1+\epsilon})[\frac{\frac{g_{11}}{(1+\epsilon)^2}\xi'_0\xi'_0 + \frac{2g_{10}}{1+\epsilon}\xi'_0 + g_{00}}{\frac{\epsilon g_{11}}{(1+\epsilon)^2}\xi'_0 + g_{10}\frac{\epsilon}{1+\epsilon}} dx^0 dx^0 + 2dx^0 dx_*^1] + (others). \tag{20}$$

The technique of conformally flat require the coefficient of $dx^0 dx^0$ in [ ] in Eq.(20) being -1

$$\frac{\frac{g_{11}\xi'_0\xi'_0 + 2g_{10}\xi'_0 + g_{00}}{\epsilon} + 2g_{00} + 2g_{10}\xi'_0 + \epsilon g_{00}}{g_{10} + g_{11}\xi'_0 + \epsilon g_{10}} = -1. \tag{21}$$

When $x^1 \mapsto \xi, \epsilon \mapsto 0$, the well-definition of the numerator of lhs. of Eq.(21) deduces

$$g_{11}\xi'_0\xi'_0 + 2g_{10}\xi'_0 + g_{00} = 0. \tag{22}$$

Now we show that the event horizon determined by (22) is not equivalent to that given by Damour-Ruffini-Zhao's method[18], which determines the location of horizon as

$$g^{11} - 2g^{1\nu}\xi'_\nu + g^{\mu\nu}\xi'_\mu\xi'_\nu = 0. \tag{23}$$

Considering the special case that $\xi'_0 = 0$, Eq.(22) simplifies to

$$g_{00}|_{x^1 \mapsto \xi} = 0 \tag{24}$$

But when $\xi'_0 = 0$, Eq.(23) simplifies

$$g^{11} - 2g^{1\nu}\xi'_\nu|_{\nu=2,3} + g^{\mu\nu}\xi'_\mu\xi'_\nu|_{\mu\nu=2,3} = 0. \tag{25}$$

Clearly Eq.(22) is not equivalent to Eq.(25) generally. Therefore the event horizons determined by the two equations are not the same in general case. Apparently the κ's are different either.

The reason is that Damour-Ruffini-Zhao's method uses $g^{\mu\nu}$ in generalized tortoise coordinates to determine ξ and κ, while technique of conformal flat uses $g_{\mu\nu}$) in generalized tortoise coordinates to determine them. Generally speaking, they do not produce the same results.

At a first glance it seems that kerr metric with term $dx_0 dx_3$ is an exception that our investigation in section two does not include. But when the metric of Kerr black hole is written in the dragging system[19][20], then it is included in our investigation.

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